Quantum Gravity induced Lorentz invariance violation in the Standard Model:hadrons

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Abstract

The most important problem of fundamental Physics is the quantization of the gravitational field. A main difficulty is the lack of available experimental tests that discriminate among the theories proposed to quantize gravity. Recently we showed that the Standard Model(SM) itself contains tiny Lorentz invariance violation(LIV) terms coming from QG. All terms depend on one arbitrary parameter α that set the scale of QG effects. In this paper we obtain the LIV for mesons and nucleons and apply it to study several effects, including the GZK anomaly.

I. INTRODUCTION

In recent years several proposal have been advanced to select theories and predict new phenomena associated to the Quantum gravitational field [1, 2, 3, 4]. Most of the new phenomenology is associated to some sort of Lorentz invariance violations(LIV's)[5, 6, 7]. Recently [8], this approach has been subjected to severe criticism.

In a previous letter[9], we asserted that the main effect of QG is to deform the measure of integration of Feynman graphs at large four momenta by a tiny LIV. The classical lagrangian is unchanged. In a similar manner, we can say that QG deforms the metric of space-time, introducing a tiny LIV proportional to (d-4)α, d being the dimension of space time in Dimensional Regularization and α is the only arbitrary parameter in the model. Such small LIV could be due to quantum fluctuations of the metric of space-time produced by QG:virtual black holes as suggested in[1], D-branes as in [10], compactification of extra-dimensions or spin-foam anisotropies [11]. A precise derivation of α will have to wait for additional progress in the available theories of QG[29]

It is possible to have modified dispersion relations without a preferred frame(DSR)[12]. Notice, however, that in our case the classical lagrangian is invariant under usual linear Lorentz transformations but not under DSR. So our LIV is more akin to radiative breaking of usual Lorentz symmetry than to DSR. Moreover the regulator R defined below and the deformed metric (5) are given in a particular inertial frame, where spatial rotational symmetry is preserved. That is why, in this paper we are ascribing to the point of view of [6] which is widely used in the literature. The preferred frame is the one where the Cosmic Background Radiation is isotropic.

Within the Standard Model, such LIV implies several remarkable effects, which are wholly

determined up to one arbitrary parameter (α).The main effects are:

The maximal attainable velocity for particles is not the speed of light, but depends on the specific couplings of the particles within the Standard Model. Noticeably, this LIV of the dispersion relations is the only acceptable, according to the very stringent bounds coming from the Ultra High Energy Cosmic Rays (UHECR) spectrum[13]. Moreover, the specific interactions between particles in the SM, determine different maximum attainable velocities for each particle, a necessary requirement to explain the Greisen[14],Zatsepin and Kuz'min[15](GZK) anomaly[6, 13, 16]. Since the Auger[17] experiment is expected to produce results in the near future, powerful tests of Lorentz invariance using the spectrum of UHECR will be available.

Also birrefringence occurs for charged leptons, but not for gauge bosons. In particular, photons and neutrinos have different maximum attainable velocities. This could be tested in the next generation of neutrino detectors such as NUBE[18, 19].

Vertices in the SM will pick up a finite LIV.

This paper is organized as follows: In chapter 2, we present the LIV cutoff regulator; section 3 contains the effect of the regulator on One Particle Irreducible Green functions(1PI); section 4 defines LIV dimensional regularization ; Explicit one loop computations are contained in section 5; the LIV for mesons and baryons is found in chapter 6; Reactions thresholds are contained in section 7; Bounds on α are derived in section 8; Section 9 contains our conclusions.

II.   CUTOFF REGULATOR

To see what are the implications of the asymmetry in the measure for renormalizable theories, we will mimic the Lorentz asymmetry of the measure by the replacement

$$\int d^d k -> \int d^d k R(\frac{k^2 + \alpha k_0^2}{\Lambda^2})$$

Here R is an arbitrary function, Λ is a cutoff with mass dimensions, that will go to infinity at the end of the calculation. We normalize $R(0) = 1$ to recover the original integral. $R(\infty) = 0$ to regulate the integral. α is a real parameter. Notice that we are assuming that rotational invariance in space is preserved. More general possibilities such as violation of rotational symmetry in space can be easily incorporated in our formalism.

This regulator has the property that for logarithmically divergent integrals, the divergent term is Lorentz invariant whereas when the cutoff goes to infinity a finite LIV part proportional to α remains.

III. ONE LOOP

Let D be the naive degree of divergence of a One Particle Irreducible (1PI) graph. The change in the measure induces modifications to the primitively log divergent integrals(D=0) In this case, the correction amounts to a finite LIV. The finite part of 1PI Green functions will not be affected. Therefore, Standard Model predictions are intact, except for the maximum attainable velocity for particles[6] and interaction vertices, which receive a finite wholly determined contribution from Quantum Gravity.

Let us analyze the primitivily divergent 1PI graphs for bosons first.

Self energy: $\chi(p) = \chi(0) + A^{\mu\nu}p_\mu p_\nu + convergent$, $A^{\mu\nu} = \frac{1}{2}\partial_\mu\partial_\nu\chi(0)$. We have:

$$A^{\mu\nu} = c_2\eta^{\mu\nu} + a^{\mu\nu}$$

c_2 is the log divergent wave function renormalization counterterm; $a^{\mu\nu}$ is a finite LIV. The on-shell condition is:

$$p^2 - m^2 - a^{\mu\nu}p_\mu p_\nu = 0$$

If spatial rotational invariance is preserved, the nonzero components of the matrix a are:

$$a^{00} = a_0; \quad a^{ii} = -a_1$$

So the maximum attainable velocity for this particle will be:

$$c_m = \sqrt{\frac{1 - a_1}{1 - a_0}} \sim 1 - (a_1 - a_0)/2 \tag{1}$$

For fermions, we have the self energy graph

$$\Sigma(p) = \Sigma(0) + s^{\mu\nu}\gamma_\nu p_\mu$$

$s^{\mu\nu}\gamma_\nu = \partial_\mu\Sigma(0)$. Moreover

$$s^{\mu\nu} = s\eta^{\mu\nu} + a^{\mu\nu}/2$$

s is a log divergent wave function renormalization counterterm; $a^{\mu\nu}$ is a finite LIV. The maximum attainable velocity of this particle will be given again by equation (1).

By doing explicit computations for all particles in the SM, we get definite predictions for the LIV, assuming a particular regulator R. However, the dependence on R amounts to a multiplicative factor. So ratios of LIV's are uniquely determined.

Vertex correction This graph has $D = 0$, so the regulator R will induce a tiny LIV.

Gauge Bosons Consider the most general quadratic Lagrangian which is gauge invariant, but could permit LIV's [30]

$$L = c^{\mu\nu\alpha\beta} F_{\mu\nu} F_{\alpha\beta}$$

$c^{\mu\nu\alpha\beta}$ is antisymmetric in $\mu\nu$ and $\alpha\beta$ and symmetric by $(\alpha, \beta) < - > (\mu, \nu)$ It implies that the most general expression for the self-energy of the gauge boson will be

$$\Pi^{\nu\beta}(p) = c^{\mu\nu\alpha\beta} p_\alpha p_\mu \Pi(p) \tag{2}$$

We see that

$$p_\nu \Pi^{\nu\beta}(p) = 0$$

$c^{\mu\nu\alpha\beta}$ is given by a logarithmically divergent integral. We get:

$$c^{\mu\nu\alpha\beta} = c_2(\eta^{\mu\alpha}\eta^{\nu\beta} - \eta^{\mu\beta}\eta^{\nu\alpha}) + a^{\mu\nu\alpha\beta} \tag{3}$$

c_2 is a Lorentz invariant counterterm and $a^{\mu\nu\alpha\beta}$ is a LIV.

It is clear that the same argument applies to massive gauge bosons that got their mass by spontaneous gauge symmetry breaking as well as to the graviton in linearized gravity.

Explicit computations are simplified by using Dimensional Regularization as explained below.

IV. LIV DIMENSIONAL REGULARIZATION

We generalize dimensional regularization to a d dimensional space with an arbitrary constant metric $g_{\mu\nu}$. We work with a positive definite metric first and then Wick rotate. We will illustrate the procedure with an example. Here $g = det(g_{\mu\nu})$ and $\Delta > 0$.

$$\frac{1}{\sqrt{g}} \int \frac{d^d k}{(2\pi)^d} \frac{k_\mu k_\nu}{(k^2 + \Delta)^n} =$$

$$\frac{1}{\sqrt{g}\Gamma(n)} \int_0^\infty dt\, t^{n-1} \int \frac{d^d k}{(2\pi)^d} k_\mu k_\nu e^{-t(g^{\alpha\beta} k_\alpha k_\beta + \Delta)} =$$

$$\frac{1}{(4\pi)^{d/2}} \frac{g_{\mu\nu}}{2} \frac{\Gamma(n-1-d/2)}{\Gamma(n)} \frac{1}{\Delta^{n-1-d/2}} \tag{4}$$

In the same manner, after Wick rotation, we obtain Appendix A4 of [18].

These definitions preserve gauge invariance, because the integration measure is invariant under shifts. To get a LIV measure, we assume that

$$g^{\mu\nu} = \eta^{\mu\nu} + (4\pi)^2 \alpha \eta^{\mu 0} \eta^{\nu 0} Res_{\epsilon=0} \tag{5}$$

where $\epsilon = 2 - \frac{d}{2}$ and $Res_{\epsilon=0}$ is the residue of the pole at $\epsilon = 0$. A formerly divergent integral will have a pole at $\epsilon = 0$, so when we take the physical limit, $\epsilon-> 0$, the answer will contain a LIV term.

That is, LIV dimensional regularization consists in:

1)Calculating the d-dimensional integrals using a general metric $g_{\mu\nu}$.

2) Gamma matrix algebra is generalized to a general metric $g_{\mu\nu}$.

3) At the end of the calculation, replace $g^{\mu\nu} = \eta^{\mu\nu} + (4\pi)^2 \alpha \eta^{\mu 0} \eta^{\nu 0} Res_{\epsilon=0}$ and then take the limit $\epsilon-> 0$.

To define the counterterms, we used the minimal substraction scheme(MSS); that is we substract the poles in ϵ from the 1PI graphs.

As a concrete example, let us evaluate a typical one loop integral that appears in the calculation of self energy graphs:

$$A^{\mu\nu} = \int \frac{d^d k}{(2\pi)^d} \frac{k^\mu k^\nu}{[k^2 - m^2 + i0]^3} = \tag{6}$$

$$\frac{i}{(4\pi)^{d/2}} \frac{g^{\mu\nu}}{2} \frac{\Gamma(2-\frac{d}{2})}{2} \frac{1}{(m^2)^{2-\frac{d}{2}}} \tag{7}$$

$$= \frac{i}{(4\pi)^{d/2}} \frac{\eta^{\mu\nu} + (4\pi)^2 \alpha \delta_0^\mu \delta_0^\nu Res_{\epsilon=0}}{2} \frac{\Gamma(2-\frac{d}{2})}{2} \frac{1}{(m^2)^{2-\frac{d}{2}}} \tag{8}$$

$$= \frac{i}{4(4\pi)^2} (\frac{\eta^{\mu\nu}}{\epsilon} + (4\pi)^2 \alpha \delta_0^\mu \delta_0^\nu) + \text{a finite LI term} \tag{9}$$

LIV Dimensional Regularization reinforces our claim that these tiny LIV's originates in Quantum Gravity. In fact the sole change of the metric of space time is a correction of order ϵ to the Minkowsky metric and this is the source of the effects studied above. Quantum Gravity is the strongest candidate to produce such effects because the gravitational field is precisely the metric of space-time and tiny LIV modifications to the flat Minkowsky metric may be produced by quantum fluctuations.

V.   EXPLICIT ONE LOOP COMPUTATIONS

A.   Example:$g\phi^3$ in six space-time dimensions

To illustrate the method using the cutoff regulator, we consider $g\phi^3$ in six space-time dimensions.

The one-loop contribution to the self energy of the particle is:

$$i\chi(q) = \frac{(-gi)^2}{2} \int \frac{d^6k}{(2\pi)^6} \frac{1}{k^2 - m^2 + i0} \frac{1}{(k-q)^2 - m^2 + i0} \tag{10}$$

The term containing the LIV is:

$$L(i\chi) = -2q^\mu q^\nu g^2 B_{\mu\nu} \tag{11}$$

$$B_{\mu\nu} = \int \frac{d^6k}{(2\pi)^6} \frac{k_\mu k_\nu}{(k^2 - m^2 + i0)^4} \tag{12}$$

To evaluate $B_{\mu\nu}$, introduce the regulator of the integration measure,

$$R = \frac{-\Lambda^2}{k^2 - \Lambda^2 + ak_0^2 + i0} \tag{13}$$

define $k = \Lambda p$ and take the limit $\Lambda - > \infty$. In this way we verify that the LIV is mass independent. Since $a << 1$, we keep only the first order in a. We end up with:

$$LB_{\mu\nu} \sim a \int \frac{d^6p}{(2\pi)^6} \frac{p_0^2 p_\mu p_\nu}{(p^2 - 1 + i0)^2 (p^2 + i0)^4} \tag{14}$$

Therefore:

$$L(i\chi) = -\frac{g^2 a q_0^2 i}{24(4\pi)^3} \tag{15}$$

B.   LIV in the Standard Model

We follow [20, 21] and use LIV Dimensional Regularization.

Photons

In the SM the photon self-energy can be written:

$$i\Pi^{\mu\nu} = i(q^2 g^{\mu\nu} - q^\mu q^\nu)(\frac{-23e^2}{48\pi^2\epsilon} + finite) \tag{16}$$

so that the LIV photon self-energy in the SM is:

$$L\Pi^{\mu\nu}(q) = -\frac{23}{3} e^2 \alpha q_\alpha q_\beta$$
$$(\eta^{\alpha\beta} \delta_0^\mu \delta_0^\nu + \eta^{\mu\nu} \delta_0^\alpha \delta_0^\beta - \eta^{\nu\beta} \delta_0^\mu \delta_0^\alpha - \eta^{\mu\alpha} \delta_0^\nu \delta_0^\beta) \tag{17}$$

It follows that the maximal attainable velocity is

$$c_\gamma = 1 - \frac{23}{6}e^2\alpha \tag{18}$$

We have included coupling to quarks and charged leptons as well as 3 generations and color.

C. Fermions

Let us consider QED, as an example. The electron self-energy to one loop is given by:

$$-i\Sigma_2(q) = (-ie)^2 \int \frac{d^d k}{(2\pi)^d} \frac{1}{\sqrt{g}} \gamma^\mu \frac{(i\slashed{k} + m)}{k^2 - m^2 + i0} \gamma_\mu \frac{-i}{(k-q)^2 - \mu^2 + i0} \tag{19}$$

To obtain the LIV, we have to evaluate (we have introduced a parameter Δ and put it to zero afterwards):

$$-iL\Sigma_2(q) = 2i(-ie)^2 \int \frac{d^d k}{(2\pi)^d} \frac{1}{\sqrt{g}} \frac{\gamma^\mu \slashed{k} \gamma_\mu k.q}{(k^2 - \Delta + i0)^3} \tag{20}$$

$$= -2i(-ie)^2(d-2) \int \frac{d^d k}{(2\pi)^d} \frac{1}{\sqrt{g}} \frac{\slashed{k} k.q}{(k^2 - \Delta + i0)^3} \tag{21}$$

$$= (-ie)^2 \frac{(d-2)}{2} \frac{1}{(4\pi)^{d/2}} \slashed{q} \frac{\Gamma(2-d/2)}{\Delta^{2-d/2}} \tag{22}$$

but

$$\slashed{q} \frac{\Gamma(2-d/2)}{\Delta^{2-d/2}} = q_\mu(\delta_a^\mu - \frac{(4\pi)^2\alpha Res_{\epsilon=0}}{2}\delta_0^\mu\delta_a^0)\gamma^a \frac{\Gamma(2-d/2)}{\Delta^{2-d/2}} = \tag{23}$$

$$\slashed{q} - \frac{(4\pi)^2\alpha}{2}q_0\gamma^0 \tag{24}$$

so,

$$-iL\Sigma_2(q) = \frac{e^2\alpha}{2}q_0\gamma^0 \tag{25}$$

Similarly, in the SM, the fermion self-energy is given by:

$$(4\pi^2)\Sigma(q) = -\frac{1}{\epsilon}\slashed{q} \sum_{graphs} (|c_V + c_A|^2 P_L + (|c_V - c_A|^2 P_R) + finite \tag{26}$$

where the fermion-gauge boson vertex is:

$$i\gamma^k(c_V - c_A\gamma^5) \tag{27}$$

and $P_L(P_R)$ are the L(R) helicity projectors.

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Therefore

$$L\Sigma(q) = \frac{\alpha}{2} q_0 \gamma^0 \sum_{graphs} (|c_V + c_A|^2 P_L + (|c_V - c_A|^2 P_R) \tag{28}$$

We apply this last result to neutrinos and charged leptons below.

Neutrinos: The maximal attainable velocity is

$$c_\nu = 1 - (3 + tan^2\theta_w)\frac{g^2\alpha}{8} \tag{29}$$

In this scenario, we predict that neutrinos [19] emitted simultaneously with photons in gamma ray bursts will not arrive simultaneously to Earth . The time delay during a flight from a source situated at a distance D will be of the order of $(5 \times 10^{-23})D/c \sim 5 \times 10^{-6}$ s, assuming $D = 10^{10}$ light-years. No dependence of the time delay on the energy of high energy photons or neutrinos should be observed(contrast with [1]). Photons will arrive earlier since $\alpha < 0$(See below). These predictions could be tested in the next generation of neutrino detectors [20].

Using R_ξ-gauges we have checked that the LIV is gauge invariant. The gauge parameter affects the Lorentz invariant part only.

Electron self-energy in the Weinberg-Salam model. Birrefringence:

Define: $e_L = \frac{1-\gamma^5}{2}e$, $e_R = \frac{1+\gamma^5}{2}e$, where e is the electron field. We get

$$c_L = 1 - (\frac{g^2}{cos^2\theta_w}(sin^2\theta_w - 1/2)^2 + e^2 + g^2/2)\frac{\alpha}{2}; \tag{30}$$

$$c_R = 1 - (e^2 + \frac{g^2 sin^4\theta_w}{cos^2\theta_w})\frac{\alpha}{2} \tag{31}$$

The difference in maximal speed for the left and right helicities is $\sim (5 \times 10^{-24})$.

VI.   MESONS AND BARYONS

In order to apply our results to the computation of the UHECR spectrum and other phenomena, we must calculate the maximal attainable velocity of hadrons. As we mentioned before, the problem is hadronization. One way to get an estimation of the effect is using effective lagrangians.

We use the results of [22, 23] for the wave function renormalization of pions and nucleons in the chiral lagrangian and Heavy Baryon Chiral Perturbation Theory. They get:

$$Z_\pi^{-1} = 1 - \frac{4m_\pi^2}{3(4\pi)^2 F^2}\frac{1}{\epsilon} + finite \tag{32}$$

$$Z_N^{-1} = 1 - \frac{9g_A^2 m_\pi^2}{4(4\pi)^2 F^2}\frac{1}{\epsilon} + finite \tag{33}$$

Here, m_π is the renormalized pion mass, F is the renormalized decay constant of pions and g_A is the axial vector coupling constant, in the chiral limit.

Using the LIV metric, we can read off the maximal attainable velocities for pions and nucleons:

$$c_\pi = 1 + \frac{2m_\pi^2 \alpha}{3F^2}$$
$$c_N = 1 + \frac{9m_\pi^2 g_A^2 \alpha}{8F^2} \tag{34}$$

VII. REACTION THRESHOLDS

Knowing the LIV for nucleons, pions, photons and electrons, we proceed to study the reactions involved in the GZK cutoff. We follow the discussion in [6, 13].

A. Photo-Pion Production $\gamma + p \to p + \pi$

Let us begin with the photo-pion production $\gamma + p \to p + \pi$. Considering the corrections provided in the dispersion relation (34) for pions and nucleons, we note that, for the photo-pion production to proceed, the following condition must be satisfied

$$2\,\delta c\, E_\pi^2 + 4E_\pi \omega \geq \frac{m_\pi^2(2m_p + m_\pi)}{m_p + m_\pi}. \tag{35}$$

where E_π is the produced pion energy and $\delta c = c_p - c_\pi$.

B. Pair Creation $\gamma + p \to p + e^+ + e^-$

Pair creation, $\gamma + p \to p + e^+ + e^-$, is greatly abundant in the sector previous to the GZK limit. When the dispersion relations for fermions are considered for both protons and electrons, it is possible to find

$$\delta c \frac{m_e}{m_p} E^2 + E\omega \geq m_e(m_p + m_e), \tag{36}$$

where E is the incident proton energy and $\delta c = c_p - c_e$.

VIII. BOUNDS

In order to study the threshold conditions (35) and (36), in the context of the GZK anomaly, we must establish some criteria. Firstly, as it is seen in [13, 24], the conventionally obtained theoretical spectrum provides a very good description of the phenomena up to an energy $\sim 4 \times 10^{19}$ eV. The main reaction taking place in this well described region is the pair creation $\gamma + p \rightarrow p + e^+ + e^-$ and, therefore, no modifications are present for this reaction up to $\sim 4 \times 10^{19}$ eV. As a consequence, and since threshold conditions offer a measure of how modified kinematics is, we will require that the threshold condition (36) for pair creation not be substantially altered by the new corrective terms.

Secondly, we have the GZK anomaly itself, which we want to explain. Since for energies greater than $\sim 8 \times 10^{19}$ eV the conventional theoretical spectrum does not fit the experimental data well, we shall require that QG corrections be able to offer a violation of the GZK-cutoff. The dominant reaction in the violated $E > 8 \times 10^{19}$ region is the photo-pion production and, therefore, we further require that the new corrective terms present in the kinematical calculations be able to shift the threshold significantly to preclude the reaction.

We begin our analysis with the threshold condition for pair production. In this case we have:

$$\delta c \frac{m_e}{m_p} E^2 + E\omega \geq m_e(m_p + m_e), \tag{37}$$

with $\delta c = c_p - c_e$. As is clear from the above condition, the minimum soft-photon energy $\omega_{\min}$ for the pair production to occur, is

$$\omega_{\min} = \frac{m_e}{E}(m_p + m_e) - \delta c \frac{m_e}{m_p} E. \tag{38}$$

It follows therefore that the condition for a significant increase or decrease in the threshold energy for pair production becomes $|\delta c| \geq m_p(m_p + m_e)/E^2$. In this way, if we do not want kinematics to be modified up to a reference energy $E_{\text{ref}} = 3 \times 10^{19}$, we must impose the following constraint

$$|c_p - c_e| < \frac{(m_p + m_e)m_p}{E_{\text{ref}}^2} = 9.8 \times 10^{-22}. \tag{39}$$

Similar treatments can be found for the analysis of other astrophysical signals like the Mkn 501 γ-rays [25], when the absence of anomalies is considered.

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Let us now consider the threshold condition for the photo-pion production. We have

$$2\,\delta c\, E_\pi^2 + 4E_\pi \omega \geq \frac{m_\pi^2(2m_p + m_\pi)}{m_p + m_\pi}. \tag{40}$$

It is possible to find that for the above condition to be violated for all energies E_π of the emerging pion, and therefore no reaction to take place, the following inequality must hold

$$c_\pi - c_p > \frac{2\omega^2(m_p + m_\pi)}{m_\pi^2(2m_p + m_\pi)} = 3.3 \times 10^{-24}\,[\omega/\omega_0]^2\,. \tag{41}$$

where $\omega_0 = KT = 2.35 \times 10^{-4}$ eV is the thermal CMBR energy.

Combining the two reactions and the standard values, $m_\pi = 139 Mev, g_A = 1.26, F = 92.4 Mev$, we get an upper and lower bound on α

$$2.2 \times 10^{-21} > -\alpha > 1.3 \times 10^{-24} \tag{42}$$

First of all, we notice that $\alpha < 0$, in order to suppress the photopion production, thus removing the GZK cutoff. This implies that photons are the fastest particles and they arrive before neutrinos coming from the same source of GRB. Moreover, photons become unstable. They decay in a electron positron pair above an energy E_0[6]. See below.

Since $c_{photon} > c_{proton}$, the strong bound of [27] is avoided: Proton is stable under Cerenkov radiation in vacuum.

If no GZK anomaly is confirmed in future experimental observations, then we should state a stronger bound for the difference $c_\pi - c_p$. Using the same assumptions to set the restriction (39) when the primordial proton reference energy is $E_{\text{ref}} = 2 \times 10^{20}$ eV, it is possible to find

$$|c_\pi - c_p| < 2.3 \times 10^{-23}. \tag{43}$$

In terms of α, this last bound may be read as

$$|\alpha| < 9.1 \times 10^{-24}, \tag{44}$$

which is a stronger bound over α than (39), offered by pair creation.

Photon unstability

It has been pointed out in [6, 27] that if $c_{photon} > c_{electron}$ then the process $\gamma \rightarrow e^+ + e^-$ is allowed above an energy E_0:

$$E_0 = m_e\sqrt{\frac{2}{\delta c}} \tag{45}$$

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where $\delta c = c_\gamma - c_e$.

In our case, we have:

$$\delta c_L = -\alpha(\frac{23}{6}e^2 - (\frac{g^2}{cos^2\theta_w}(sin^2\theta_w - 1/2)^2 + e^2 + g^2/2)/2) \tag{46}$$

$$\delta c_R = -\alpha(\frac{23}{6}e^2 - (e^2 + \frac{g^2 sin^4\theta_w}{cos^2\theta_w})/2) \tag{47}$$

Therefore, with

$$EL_0 = 2.3 \times 10^8 Gev$$

$$ER_0 = 1.9 \times 10^8 Gev \tag{48}$$

So, we should not detect photons with energies above $2.3 \times 10^8 Gev$

Neutral pion Stability

Following [6] we study the main decay process of neutral pion $\pi_0 \to \gamma + \gamma$ .This becomes possible if $c_\gamma > c_\pi$ and above an energy

$$E_\pi = \frac{m_\pi}{\sqrt{2(c_\gamma - c_\pi)}} \tag{49}$$

Using the bound $c_\gamma - c_\pi < 10^{-22}$ obtained in [28], we get

$$|\alpha| < 5.4 \times 10^{-23} \tag{50}$$

In our numerical estimates we have chosen $\alpha = -5 \times 10^{-23}$.

We get $E_\pi = 10^{19}eV$. Therefore we expect that neutral pions above this energy are stable, so they could be a primary component of UHECR. Photons will be unstable above this energy by the same mechanism. Notice however that photons are unstable at a lower energy due to electron-positron pair creation (48).

IX. CONCLUSIONS

In this paper we have computed the LIV induced by Quantum Gravity on Baryons and Mesons, using the Chiral Lagrangian approach. This permitted to fix that $\alpha < 0$, in order to explain the GZK anomaly. Studying several available processes, we found bounds on α:

From pair creation and absence of photopion creation: $2.2 \times 10^{-21} > -\alpha > 1.3 \times 10^{-24}$.

From pion stability and the most stringent experimental bound found in [28]: $|\alpha| < 5.4 \times 10^{-23}$.

Then, several predictions are obtained:Photons are unstable above an energy $2.3 \times 10^8 Gev$.

Neutral pions are stable above an energy $E_\pi = 10^{19} eV$; so they could be a primary component of UHECR, thus evading the GZK cutoff.

Moreover, in time of flight experiments, photons will arrive before neutrinos, assuming that they were emitted simultaneously at the source. No energy dependence of the time delay should be observed. The time delay during a flight from a source situated at a distance D will be of the order of $(5 \times 10^{-23})D/c \sim 5 \times 10^{-6}$ s, assuming $D = 10^{10}$ light-years.

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[29] Such derivation must explain why the LIV parameter is so small. Progress in this direction is

in [3, 4, 13]. There α appears as $(l_P/L)^2$, where l_P is Planck's lenght and L is defined by the semiclassical gravitational state in Loop Quantum Gravity. If $L \sim 10^{11} l_P$, an α of the right order is obtained

[30] A Chern-Simons term is absent due to the symmetry $k_\mu - > -k_\mu$, which is preserved by the regulator.